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Generalized Dirichlet Branes and Zero-modes

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Abstract

We investigate the effective dynamics of an arbitrary Dirichlet p-brane, in a path-integral formalism, by incorporating the massless excitations of closed string modes in open bosonic string theory. It is shown that the closed string background fields in the bosonic sector of type IIB induce invariant extrinsic curvature on the world-volume. In addition, the curvature can be seen to be associated with a divergence at the boundary of the string world-sheet. The renormalization of the collective coordinates, next to the leading order in its derivative expansion, is performed to handle the divergence and the effective dynamics is encoded in the Dirac-Born-Infeld action. Futhermore, the collective dynamics is generalized to include the appropriate fermionic partners in type I super-string theory. The role of zero modes is reviewed in terms of the collective coordinates and the gauge theory on the world-voulme is argued to be non-local in presence of the $U(1)$ invariant field.

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1 Introduction

In super-string theory, most of the non-perturbative insights have been analyzed by investigating various features of Dirichlet p-brane (D_p-brane), where 'p' denotes the number of spatial dimensions. Since strings are coupled to the non-trivial background fields through conformal symmetry, a systematic study [1] of the D_p-brane involves arbitrary type II string backgrounds and have been in focus since its inception as the Ramond-Ramond (RR) charge carriers [2]. Among various aspects of the D_p-brane, one of the important is the D_p-brane dynamics. Several investigations in this direction have been discussed using different techniques [3]. In general, the world-volume gauge theory can be seen as an approximation to the underlying open string theory [4, 5, 6, 7, 8]. In this context, a path-integral formalism to deal with the collective dynamics of a D-particle (D_0) [9] and subsequently for a D-string (D_1) [10, 11] were developed.

From a microscopic point of view, the D_p-brane is characterized by the $(p+1)$-dimensional hyper-surfaces with open string ending on it [4]. Thus, the $U(1)$ gauge field associated with the ends of the open string contribute towards the world-volume dynamics of the D_p-branes. In fact, the world-volume gauge field gives rise to the extra degrees of freedom unlike the Green-Schwarz (fundamental) string. These extra states are essentially the massive states of the open string and can be seen to be responsible for the non-local description on the world-voulme of the D_p-branes. To a tree level approximation in string theory, the gauge theory on the world-volume of a D_p-brane is a Yang-Mills theory. The next order in α' expansion gives rise to the Born-Infeld action [5] and describes the world-volume dynamics of the D_p-branes. In general, the Born-Infeld action incorporates corrections to all orders in α' and the exact dynamics of a D_p-brane still remains unanswered. Though D_p-branes are BPS objects in string theory, in most of the investigations related to its dynamics, bosonic branes are considered. Nevertheless, the formulation of the super-symmetric D_p-brane dynamics play an important role and has been worked out [12] in great detail in the type II closed string theories.

In this paper, we extend our previous analysis [11] for a D-string ($p = 1$) in presence of closed string backgrounds to an arbitrary D_p-brane. In addition, we consider the appropriate super-partners in the present context and study the D_p-brane world-volume geometry. We perform the analysis for the D_p-branes in type IIA or IIB[2] super-string theory where they play a natural role due to their RR charges. For definiteness, we consider a D_p-brane, with odd 'p' ($= 1, 3, 5, \ldots$) in the bosonic sectors (NS-NS and RR) of type IIB closed string theory. The result for the even 'p' ($= 0, 2, 4 \ldots$) D_p-brane of type IIA can be formally obtained from that of type IIB D_p-brane

[2] 'p' takes even value for type IIA and odd in case of type IIB.

by screening the Kalb-Ramond (KR) potential in the NS-NS sector. It can be seen that the induced metric gives rise to the curvature on the world-volume. In case of type IIB super-string, the KR potential can also be seen to induce additional curvature on the world-volume of the D_p-brane. In any case (even or odd 'p'), the induced (extrinsic) curvature on the world-volume of the D_p-brane and makes it a curved one.

We plan to present an explicit computation for an arbitrary bosonic D_p-brane in (oriented) open string theory. The duality of the world-sheet can be used to interprete the open string result in the type II closed string theories. We take into account the derivative expansion of the collective coordinates next to leading order in α' and the effective dynamics can be seen to be that of the Dirac-Born-Infeld (DBI). In fact, the field contents in the NS-NS sector of the type IIB closed string and that of the oriented open bosonic string are identical. The remaining fields in the RR sector act as sources for the D_p-brane and determine its charges. Obtaining the effective dynamics for the bosonic D_p-brane, we generalize the formalism to a super-symmetric case by considering appropriate fermionic partners for open string coordinates as well as for the collective coordinates. To perform the computations, we consider type I open super-string (un-oriented, $N = 1$ super-symmetry) [13, 14], and present necessary steps for the fermionic part to make the presentation concise. In other words, we investigate the wolrd-volume dynamics of an arbitrary D_p-brane, with the generic type II string backgrounds, in open string channel. The presence of background fields can be seen to be a deformation [15] of the D_p-brane world-volume and the non-commutative geometry [16, 17] shows up naturally at the boundary of the open strings [18]. We investigate the induce geometry due to the zero modes of the open string at its boundary and conclude with a note on the non-local description of gauge theory on the D_p-brane world-volume.

2 Open string fluctuations on a p-brane

It is known that the effective dynamics of a D_p-brane is due to the fluctuations governed by the end of the open super-string (whose world-sheet is a disk) with appropriate boundary conditions. For instance, Dirichlet boundary conditions in the transverse directions define the position of the D_p-brane and the remaining $(p + 1)$-directions satisfy the Neumann conditions along the world-volume.

To begin with, consider an arbitrary D_p-brane ($p = 1, 2, \ldots$), characterized by its space-time coordinates $f^\mu(t, \sigma_i)$, for $(i = 1, \ldots, p)$ with Lorentzian signature $(-, +, +, \ldots +)$, in presence of closed string background fields. The bosonic D_p-brane sub-manifold (world-volume) may be seen as an embedding in space-time ($\mu = 0, 1, \ldots, p, p+1, \ldots, 25$). At the disk boundary[3] $\partial\Sigma$, the Lorentz covariant condition becomes [4]

$$X^\mu(\theta) \;=\; f^\mu(t(\theta), \sigma_i(\theta)) \;, \tag{1}$$

where $X^\mu(z, \bar{z})$ is the open string coordinates parameterized by the polar angle θ with $(0 \leq \theta \leq 2\pi)$ on $\partial\Sigma$.

The interaction of the D_p-brane (for odd 'p') with the massless excitations of closed string; namely metric, $G_{\mu\nu}$, KR antisymmetric two-form potential,[4] $B_{\mu\nu}$ and the dilaton, Φ, can be described in open string theory by a non-trivial generalization of our earlier formulations [10, 11] for a D-string. It is known that the open string possesses a $U(1)$ gauge field $A_\mu(X)$ in space-time and can also be seen as a requirement for the consistency of the closed string KR potential [19]. The background gauge field interacts with the open string at its boundary where as the closed string background fields interact in bulk. Then the non-linear sigma model describing the dynamics of the open string can be written [20] for a constant dilaton, in a conformal gauge, as

$$\begin{aligned} S[X, A, B] \;=\; \frac{1}{4\pi\alpha'} \int_\Sigma d^2z \Big(G_{\mu\nu}(X)\partial_{\bar{a}}X^\mu \partial^{\bar{a}}X^\nu \;-\; i\,\epsilon^{\bar{a}\bar{b}}\, B_{\mu\nu}(X)\partial_{\bar{a}}X^\mu \partial_{\bar{b}}X^\nu \Big) \\ +\, i \int_{\partial\Sigma} d\theta\; G_{\mu\nu}(X)A^\mu(X)\; \partial_\theta X^\nu \;. \end{aligned} \tag{2}$$

The effective dynamics for the D_p-branes can be taken into account by generalizing the path-integral [21, 9, 10] in presence of the curved background fields [11]. In fact, we make use of the Polyakov path-integral formulation [22] to describe the generalized dynamics of the D_p-branes. At low energy, the effective action for the D_p-brane can be obtained from the renormalized non-linear sigma model partition function which is the disk amplitude modulo the massless closed string vertex operator. In this formalism, the effective dynamics can be generalized from refs.[10, 11] and becomes

$$\mathcal{S}_{\rm eff}\Big(f, A, B\Big) \;= \frac{1}{g_s}\int \mathcal{D}X^\mu(z)\mathcal{D}t(\theta)\; \mathcal{D}\sigma_i(\theta)\; \delta\Big(\; X^\mu(\theta) - f^\mu(t(\theta), \sigma_i(\theta))\;\Big) \cdot \exp\Big(- S[X, A, B] \;\Big) \;, \tag{3}$$

where $g_s = e^\phi$ is the closed string coupling constant. The δ-function in eq.(3) takes care of the Dirichlet boundary conditions and hence responsible for the D_p-brane description. The path-integral (3) needs to be evaluated in bulk as well as at the boundary, to obtain non-linear dynamics of a D_p-brane.

[3] For convenience, we compute for the bosonic string. Appropriate fermionic partners shall be introduced at a later point in section 8.

[4] For even 'p', similar analysis holds by setting $B_{\mu\nu} = 0$ in the NS-NS sector (type IIA),which is the case for non-oriented open string.

3 Curvatures on the D_p-brane world-volume

The background fields can be seen to induce curvatures on the $(p+1)$-dimensional world-volume of a D_p-brane. The induced fields can be expressed in terms of the D_p-brane collective coordinates, $f^\mu(t,\sigma_i)$, and become

$$\begin{aligned} h_{ab} &= G_{\mu\nu}(X)\ \partial_a f^\mu \partial_b f^\nu \ , \\ B_{ab} &= B_{\mu\nu}(X)\ \partial_a f^\mu \partial_b f^\nu \\ \text{and} \qquad C_{abc\ldots} &= C_{\mu\nu\rho\ldots}(X)\ \partial_a f^\mu \partial_b f^\nu \partial_c f^\rho \ldots \ , \end{aligned} \tag{4}$$

where h_{ab}, B_{ab} and $C_{abc\ldots}$ for $(a,b,c,\ldots = 0,1,2\ldots,p)$ are respectively the metric, KR two-form and the RR $(p+1)$-form induced on the D_p-brane. This in turn leads to the extrinsic curvature, K, on the D_p-brane world-volume and makes it a curved one. The induced RR forms mixes with the KR form in a gauge invariant way and the source term describes a $(p+2)$-dimensional sub-manifold with its boundary representing the world-volume of the D_p-brane. For a generalized D_p-brane, the boundary value of the Wess-Zumino (WZ)-action can be given by [23]

$$S_{WZ}(f,C) \ = \ Q_p \int dt\ d^p\sigma\ e^{(B+\bar{F})} \wedge C \ , \tag{5}$$

where Q_p denotes the p-brane charge density and the RR potential, C, takes the appropriate forms with the invariant $U(1)$ field strengths.[5] The extrinsic curvature of a D_p-brane in presence of arbitrary backgrounds of type II string theory depends on the induced metric and KR two-form fields and can be expressed as

$$\begin{aligned} K^\mu_{ab} &= P^{\mu\nu}\ \partial_a \partial_b f_\nu \ , \\ K^\mu_{abc} &= \partial_a\partial_b\partial_c f^\mu - \partial_a \Gamma^\mu{}_{bc} + 2\Gamma^d{}_{ab}\Gamma^\mu{}_{dc} - \frac{1}{2}\partial_a H^\mu{}_{bc} + H^d{}_{ab} H^\mu{}_{dc} \ , \end{aligned} \tag{6}$$

where Γ is the Christoffel connection and H $(= dB)$ denotes the field strength corresponding to the KR two-form. $P^{\mu\nu}$ denotes the normal projected space and can be expressed in terms of the tangential space $h^{\mu\nu}$:

$$\begin{aligned} & P^{\mu\nu} = G^{\mu\nu} + B^{\mu\nu} - h^{\mu\nu} \ , \\ \text{where} \qquad & h^{\mu\nu} = \left(h^{ab} + B^{ab}\right) \partial_a f^\mu \partial_b f^\nu \ . \end{aligned} \tag{7}$$

In order to perform the boundary path-integrals, the brane coordinates $f^\mu(t(\theta),\sigma_i(\theta))$ can be re-written in its derivative expansion around its center of mass coordinate $f^\mu(t,\sigma_i)$. The appro-

[5]The effective action obtained by path-integrating eq.(3) along with the WZ action (5) play a vital role in the super-symmetric formulation

priate geodesic expansion [24] for the $(p+1)$-dimensional world-volume becomes

$$f^{\mu}\left(\,t(\theta),\sigma_i(\theta)\,\right) \;=\; f^{\mu}(t,\sigma_i) \;+\; \partial_a f^{\mu}\Big(t(\sigma),\sigma_i(\theta)\Big)\zeta^a(\theta) \;+\; \frac{1}{2}K^{\mu}_{ab}\zeta^a(\theta)\zeta^b(\theta) \\ +\; \frac{1}{3!}K^{\mu}_{abc}\zeta^a(\theta)\zeta^b(\theta)\zeta^c(\theta) \;+\; \mathcal{O}(\zeta^4)\;, \qquad (8)$$

where $\zeta^a(\theta)$ are the normal coordinates defined on the world-volume of a p-brane. The expansion for a p-brane takes the similar form as the case of D-string [10]. However the explicit form of the curvature (6) is much involved with geometry due to the additional three-form field strength H_{abc}.

To leading order in the expansion (8), the boundary of the disk is mapped on to a point (t,σ_i) on the $(p+1)$-dimensional world-volume. Then, an orthonormal frame [9, 10, 11] can be set up at that point to simplify the computation. As a consequence, the sub-leading terms in eq.(8), become the quantum fluctuations in this frame. In addition, the basis vectors, $\hat{e}^{\mu}_{a}$, $(a\;=\;0,1,\ldots p)$ span the $(p+1)$-dimensional tangential space and satisfy the Neumann boundary conditions. The remaining unit vectors, $\hat{e}^{\mu}_{\alpha}$, $(\alpha\;=\;p+1,\ldots,25)$ lie in a transverse space with Dirichlet boundary conditions. They can be expressed for $A=(a,\alpha)$ as

$$\begin{aligned} \hat{e}^{\mu}_{A} &= \mathcal{N}_{(A)}\; e^{\mu}_{A}\;, \\ \hat{e}^{\mu}_{a} &= \mathcal{N}_{(a)}\partial_a f^{\mu}\;, \\ G^{\mu\nu} &= \hat{e}^{\mu}_{A}\hat{e}^{\nu}_{B}\;\eta^{AB}\;, \\ B^{\mu\nu} &= \hat{e}^{\mu}_{A}\hat{e}^{\nu}_{B}\mathcal{E}^{AB}\;, \\ h^{\mu\nu} &= \hat{e}^{\mu}_{a}\hat{e}^{\nu a} \;+\; B^{ab}\;\hat{e}^{\mu}_{a}\hat{e}^{\nu}_{b} \\ P^{\mu\nu} &= \hat{e}^{\mu}_{\alpha}\hat{e}^{\nu\alpha} \;+\; B^{\alpha\beta}\;\hat{e}^{\mu}_{\alpha}\hat{e}^{\nu}_{\beta}\;, \end{aligned} \qquad (9)$$

where η_{AB} and $\mathcal{E}_{AB}$ are the flat backgrounds representing the Minkowskian metric and the KR two-form respectively. $\mathcal{N}_{(a)}$ are the (induced) metric dependent normalizations in the tangential space and satisfy

$$\sum_{(a\neq b)=0}^{p} \mathcal{N}^{-2}_{(a)}\mathcal{N}^{-2}_{(b)} \;=\; -\;h\;, \qquad (10)$$

where $h\;=\;\det h_{ab}$ and the normalizations in the transverse space are denoted as $\mathcal{N}_{(\alpha)}\;=\;(1,1,\ldots 1)$.

4 Boundary conditions for non-zero modes

In order to simplify the computation, we separate out the zero mode x^{μ} from the string coordinate, $X^{\mu}(z,\bar{z})\;=\;x^{\mu}\;+\;\xi^{\mu}(z,\bar{z})$. Then the non-zero mode ξ^{μ} can be expressed in terms of the orthonormal coordinates $\rho^A(z,\bar{z})$:

$$\xi^\mu(z,\bar{z}) \;=\; \sum_A \hat{e}^\mu_A \;\rho^A(z,\bar{z}) \;, \tag{11}$$

where $\rho_a(z,\bar{z})$ and $\rho_\alpha(z,\bar{z})$ correspond to the components in tangential and transverse spaces respectively.

The background fields, namely metric $G_{\mu\nu}(X)$, KR two-form $B_{\mu\nu}(X)$ and the $U(1)$ gauge field $A_\mu(X)$ can also be expanded around their zero-modes:

$$\begin{aligned}
G_{\mu\nu}(X) &= \eta_{\mu\nu} + \partial_\lambda G_{\mu\nu}\,\xi^\lambda + \ldots\;, \\
B_{\mu\nu}(X) &= \epsilon_{\mu\nu} + \partial_\lambda B_{\mu\nu}\,\xi^\lambda + \ldots \\
\text{and}\qquad A_\mu(X) &= a_\mu + \frac{1}{2}F_{\mu\nu}\,\xi^\nu + \ldots\;,
\end{aligned} \tag{12}$$

where $\eta_{\mu\nu}$ is the flat metric. $\epsilon_{\mu\nu}$ and a_μ are the constant modes for the antisymmetric two-form and gauge field respectively. For a constant field strength F_{ab} on the $(p+1)$-dimensional world-volume, the non-linear sigma model action (2) simplifies drastically in the orthonormal frame and becomes [11]

$$\begin{aligned}
S[\rho,B,A] \;=\; \frac{1}{4\pi\alpha'}\Big[-\int_\Sigma d^2z\;\rho_A\partial^2\rho^A \;+\; \int_{\partial\Sigma} d\theta\;\Big(\;\rho_A\partial_n\rho^A \\
+\; i\,\mathcal{N}_{(A)}\mathcal{N}_{(B)}\;\left(B_{AB}+\bar{F}_{AB}\;\right)\;\rho^A\partial_\theta\rho^B\;\Big)\;,
\end{aligned} \tag{13}$$

where ∂_n denotes the normal derivative, $B_{AB} \equiv B_{\mu\nu}e^\mu_A e^\nu_B$, $\bar{F}_{AB} \equiv 2\pi\alpha'\;F_{AB}$, $\partial_A \equiv e^\nu_A\nabla_\nu$ and $A_B \equiv e^\mu_B A_\mu$. Then the boundary conditions for the non-zero modes of the open string can be derived from the above eq.(13) and can be given as

$$\begin{aligned}
\partial_n\rho_a(\theta) \;+\; i\,\mathcal{N}_{(a)}\mathcal{N}_{(b)}\;\left(B_{ab}+\bar{F}_{ab}\right)\;\partial_\theta\rho^b(\theta) \;&=\; 0 \\
\text{and}\qquad \rho_\alpha(\theta) \;&=\; 0\;,
\end{aligned} \tag{14}$$

where $(B_{ab}+\bar{F}_{ab})$ is the invariant $U(1)$ field strength on the world-volume of an arbitrary D_p-brane.[6]

5 Integration over string modes

The effective dynamics of the D_p-brane can be obtained by performing the path-integrations in bulk as well as over the boundary fields. Let us consider the path-integral over the string modes

[6] In section 7, we analyze the boundary conditions to view some of the interesting features of the world-volume dynamics

$\rho_A(z,\bar{z})$ uniformly in bulk. In the orthonormal frame, the path-integral can be dealt separately over the transverse and the longitudinal components. The path-integral over the transverse components, ρ_α, is a Gaussian and can be seen to be trivial using the Dirichlet boundary conditions (14). Thus the computation is essentially reduced to that over the longitudinal components ρ_a. It can be expressed as

$$I^L_\rho \;=\; \int \mathcal{D}\rho_a \;\exp\Big(-\frac{1}{4\pi\alpha'}\int_\Sigma d^2z\,\Big[\,\partial_{\bar{a}}\rho^a\partial^{\bar{a}}\rho_a \;-\; \mathcal{N}_{(a)}\mathcal{N}_{(b)}\Big(B_{ab}+\bar{F}_{ab}\Big) \cdot\delta(|z|-1)\;\rho^a\partial_z\rho^b\Big]\Big)\;\cdot\exp\Big(\,i\int_\Sigma d^2z\;\delta(|z|-1)\;\nu_a(z)\;\rho^a(z)\,\Big)\;, \qquad (15)$$

where $\delta(|z|-1)\;\nu_a(z)=\nu_a(\theta)$ denote the Lagrange multiplier fields due to the boundary conditions in eq.(1). The above Gaussian integral (15) is straightforward to perform and one obtains

$$I^L_\rho = \;[\text{Jacobian}]\;\cdot\exp\Big(\,\frac{\alpha'}{2}\int d\theta\;d\theta'\;\nu_a(\theta)\;G_{ab}(\theta,\theta')\;\nu_b(\theta')\;\eta_{ab}\,\Big)\;, \qquad (16)$$

where $G_{ab}(\theta,\theta')$ denotes the Neumann propagator on the boundary of an unit disk. The matrix propagator satisfies

$$\begin{aligned} &\partial^2 G_{ab}(z,z') \;=\; 2\pi\;\eta_{ab}\;\delta^{(2)}(z,z') && \text{in bulk}\\ \text{and}\quad &\partial_n G_{ab}(z,z') \;+\; i\,\mathcal{N}_{(a)}\mathcal{N}_{(b)}\;\left(B_{ab}+\bar{F}_{ab}\right)\;\partial_\theta G_{ab}(z,z') \;=\; 0 && \text{on } \partial\Sigma\;. \end{aligned} \qquad (17)$$

Finally, the expressions in eq.(17) are analyzed and the explicit form for the $(p+1)$ dimensional square (orthogonal) matrix propagator can be expressed as

$$G_{ab}(z,z') \;=\; \eta_{ab}\;\ln|z-z'| \;+\; \frac{1}{2}\left(\frac{1-\mathcal{N}_{(a)}\mathcal{N}_{(b)}\;(B+\bar{F})}{1+\mathcal{N}_{(a)}\mathcal{N}_{(b)}\;(B+\bar{F})}\right)_{ab}\;\ln\left(1-\frac{1}{z\bar{z}'}\right) \;+\; \frac{1}{2}\left(\frac{1+\mathcal{N}_{(a)}\mathcal{N}_{(b)}\;(B+\bar{F})}{1-\mathcal{N}_{(a)}\mathcal{N}_{(b)}\;(B+\bar{F})}\right)_{ab}\;\ln\left(1-\frac{1}{z'\bar{z}}\right)\;. \qquad (18)$$

The effect of the $U(1)$ gauge field can be seen as a Lorentz rotation with respect to the one of vanishing gauge field. On the boundary $\partial\Sigma$, the diagonal part of the propagator matrix diverges as $\theta\to\theta'$. We regularize the propagator by introducing a cut off ϵ [13]

$$G_{aa}(\theta,\theta') \;=\; -2\;\delta_{aa}\;h\;\left[h-\det\left(B_{ab}+\bar{F}_{ab}\right)\right]^{-1}\sum_{n=1}^{\infty}\frac{e^{-\epsilon n}}{n}\cos\;n(\theta-\theta')\;. \qquad (19)$$

In the limit $\theta'\to\theta$, the propagator $G_{aa}(\theta,\theta)$ can be seen to contain a divergence and is given

$$G_{aa}(\theta,\theta) \;=\; 2\delta_{aa}\;h\;\left[h-\det\left(B_{ab}+\bar{F}_{ab}\right)\right]^{-1}\;\ln\;\epsilon\;. \qquad (20)$$

On the other hand, the Jacobian in eq.(16) can be written as

$$[\text{Jacobian}] \;=\; \Big(-\det\Big[\eta_{ab}\,\partial^2 \;+\; \mathcal{N}_{(a)}\mathcal{N}_{(b)}\,(B_{ab}+\bar{F}_{ab})\,\delta(|z|-1)\partial_z\Big]\Big)^{-\frac{1}{2}} \,. \tag{21}$$

Using the Fourier mode expansion on a boundary circle, the Jacobian can be re-expressed as

$$[\text{Jacobian}] \;=\; \prod_{n=1}^{\infty}\left[\,1 \;-\; \sum_{a,b=0}^{p}\,\mathcal{N}_{(a)}^2\mathcal{N}_{(b)}^2\;\det\left(B_{ab}+\bar{F}_{ab}\right)\right]^{-1} \,. \tag{22}$$

The zeta-function regularization can be performed and one obtains

$$\begin{aligned} -\sum_{n=1}^{\infty} 1 \;&=\; -\lim_{q\to 0}\sum_{n=1}^{\infty} n^{-q} \\ &=\; \zeta(0)\;. \end{aligned} \tag{23}$$

Finally, the Jacobian for the path-integral in bulk reduces to a simple form:

$$[\text{Jacobian}] \;=\; \frac{1}{\sqrt{h}}\;\left(\,h\;+B+\bar{F}\,\right)^{\frac{1}{2}} \;, \tag{24}$$

where $h+B+\bar{F} \;=\; \det(h_{ab}+B_{ab}+\bar{F}_{ab})$. Since the computation of disk amplitude is modulo the massless closed string modes, the Jacobian obtained (24) is the only contribution from the bulk as a whole and corresponds to an exact (in α') result.

6 Integration over boundary fields

Now, we are in a position to perform the path-integral over the boundary fields $\nu_a(\theta)$ and $\zeta^a(\theta)$ by assembling the relevant terms. The path-integral over the Lagrange multiplier field $\nu_a(\theta)$ can be expressed as an Gaussian and thus straight-forward to perform. The complexity is due to the curvatures and the source term can be explicitly expressed as

$$\begin{aligned} J_a(\theta) \;=\; \mathcal{N}_{(a)}^{-1}\,\eta_{ab}\Big(\,\zeta^b(\theta) \;-\; \frac{1}{3!}\Big[\;&K^{\mu}_{lm}K_{\mu np}\,h^{bp} \;+\; \Gamma^b_{lp}\Gamma^p_{mn} \;+\; \frac{1}{4}H^b_{lp}H^p_{mn} \\ &+\;\partial_l\partial_m f^{\mu}\;\partial_q f_{\mu}\;\partial_n h^{bq} + \;2\partial_l\partial_m f^{\mu}\partial_q f_{\mu}\Gamma^q_{np}h^{bp} \\ &+\;\partial_l\partial_m f^{\mu}\;\partial_q f_{\mu}\;H^q_{np}\;h^{bp} \;-\; \partial_r f^{\mu}\partial_q f_{\mu}\;\Gamma^q_{np}\Gamma^r_{lm}\;h^{bp} \\ &-\;\frac{1}{4}\partial_r f^{\mu}\partial_q f_{\mu}\;H^q_{np}H^r_{lm}\;h^{bp}\;\Big]\;\zeta^l(\theta)\zeta^m(\theta)\zeta^n(\theta)\,\Big) \;+\; \mathcal{O}(\zeta^4)\;. \end{aligned} \tag{25}$$

The result of the $\nu_a(\theta)$-integration can be given

$$I_\nu \;\equiv\; \exp\left(\frac{1}{2\alpha'}\int d\theta\; d\theta' J_a(\theta)\;G^{-1}_{ab}(\theta,\theta')\;J_b(\theta')\;\eta_{ab}\right)\;. \tag{26}$$

As can be analyzed from the source term (25), to $\mathcal{O}(\alpha')$, the $\zeta_a(\theta)$-integral contains a quartic interaction term apart from the quadratic part. In order to simplify the calculation, we make use of a re-scaling [10] for an arbitrary p-brane :

$$\zeta^a(\theta) \ = \ \sqrt{\alpha'}\ \mathcal{N}_{(a)}\ \bar{\zeta}^a(\theta)\ . \tag{27}$$

The corresponding change in the functional measure is calculated by using the Fourier mode expansion and the zeta-function regularization. It can be expressed as

$$\mathcal{D}\zeta^a(\theta) \ = \ \alpha'^{-\frac{(p+1)}{2}} \sum_{(a\neq b)=0}^{p} \mathcal{N}_{(a)}^{-1}\ \mathcal{N}_{(b)}^{-1}\ \mathcal{D}\bar{\zeta}^a(\theta)\ . \tag{28}$$

The quartic interaction term in the $\zeta^a(\theta)$-integral can be simplified drastically by using the propagator

$$\bar{\zeta}^a(\theta)\bar{\zeta}^b(\theta') \ = \ \eta^{ab}\ G(\theta,\theta')\ . \tag{29}$$

After some calculations, the path-integrated boundary part can be given by

$$I_{\partial\Sigma} \equiv (\alpha')^{-\frac{(p+1)}{2}}\sqrt{-h}\ \Big(1-\alpha'\ h\left[h-\det\left(B_{ab}+\bar{F}_{ab}\right)\right]^{-1}\mathcal{N}_{(a)}^2\eta_{aa}\ K^{\lambda}_{aa}\ K_{\lambda ab}\ h^{ab}\ln\epsilon+\mathcal{O}(\alpha'^2)\Big)\ . \tag{30}$$

Now the effective dynamics of a D_p-brane can be obtained by considering the path-integrated results in bulk (24) and boundary (30) along with the zero-modes contribution as a volume integral. Considering all the factors properly, the disk amplitude becomes

$$\begin{aligned}\mathcal{S}_{\rm eff}(f,A,B) \ &= \ T_p \int\ dt\ d^p\sigma\ \sqrt{-\det\Big(\ h+B+\bar{F}\ \Big)}\\ &\cdot\Big(1-\alpha'\ h\ \left[h-\det\left(B_{ab}+\bar{F}_{ab}\right)\right]^{-1}\mathcal{N}_{(a)}^2\ \eta_{aa}\ K^{\lambda}_{aa}\ K_{\lambda ab}h^{ab}\ \ln\epsilon\ +\ \mathcal{O}(\alpha'^2)\Big)\ ,\end{aligned} \tag{31}$$

where $T_p = 1/[g_s(\alpha')^{\frac{(p+1)}{2}}]$ denotes the D_p-brane tension. The above sub-leading term is due to the extrinsic curvature, K, and is associated with a divergence ($\ln\epsilon$). The singularity can be isolated by re-normalization of the string tension and does not affect the formulation. The corrections can be absorbed by mass re-normalization and essentially correspond to the D_p-brane world-volume re-normalization. To obtain a renormalized amplitude, we re-define the D_p-brane world-volume $f^\mu = f^\mu_R + \sum_a \partial_a f^\mu_R$ with the divergent term:

$$\delta_a f^\mu_R \ = \ -\ \alpha'\ \left(\frac{h_R+\det\Big(B^R_{ab}+\bar{F}_{ab}\Big)}{h_R-\det\left(B^R_{ab}+\bar{F}_{ab}\right)}\right)\ \eta_{aa}\ \mathcal{N}_{(a)}^2\ K^{\mu}_{aa}\ \ln\epsilon\ . \tag{32}$$

The index R stands for the re-normalization of the D_p-brane. Then the effective dynamics (31) for a curved D_p-brane ($p = 1, 3, 5 \ldots$), next to leading order, becomes precisely the Dirac-Born-Infeld (DBI) action

$$\mathcal{S}_{\rm eff}(f_R, A, B_R) \;=\; T_p \int dt\; d^p\sigma\; \sqrt{-\det\Big(\; h_R \;+\; B_R \;+\; \bar{F}\;\Big)} \;+\; \mathcal{O}(\alpha'^2)\ . \tag{33}$$

Thus the effective low energy dynamics of a curved D_p-brane, next to leading order in its derivative expansion is described by the DBI action. Since the computation is a low energy approximation, the renormalized DBI action (33) receives corrections from all the higher orders in α' which in turn is associated with the derivative expansion of the D_p-brane coordinates $f^\mu(t, \sigma_i)$ in this formalism.

7 Zero-modes and world-volume geometry

In this section, we re-call the boundary conditions (14), for the non-zero modes of open string, derived in section 4. The tangential components of the string coordinates, $\rho_a(\theta)$, can be re-written and the boundary conditions for a D_p-brane become

$$\begin{aligned}
&\partial_n\rho_0 \;+\; i\,\mathcal{N}_{(0)}\,\mathcal{N}_{(i)}E_i\;\partial_\theta\rho^i \;=\; 0\ ,\\
&\partial_n\rho_i \;-\; i\,\mathcal{N}_{(0)}\,\mathcal{N}_{(i)}E_i\;\partial_\theta\rho^0 \;=\; 0\\
\text{and}\qquad &\partial_n\rho_i \;+\; i\,\mathcal{N}_{(i)}\,\mathcal{N}_{(j)}(B_{ij}+\bar{F}_{ij})\;\partial_\theta\rho^j \;=\; 0\ ,
\end{aligned} \tag{34}$$

where $E_i = (B_{0i} + \bar{F}_{0i})$ corresponds to the electric field components $(E_1, E_2, \ldots E_p)$ and $(B_{ij} + \bar{F}_{ij})$ defines the magnetic part of the background fields. For membranes and higher dimensional branes, both the electric and magnetic fields are non-vanishing unlike the string. In the orthogonal moving frame, the canonical momenta conjugate to (ρ^a, ρ^α) can be expressed as

$$\begin{aligned}
P^a(z) \;&=\; \partial_n\rho^a \;+\; \partial_\theta\rho^b(B^a_b + \bar{F}^a_b)\\
\text{and}\qquad P^\alpha(z) \;&=\; \partial_n\rho^\alpha\ .
\end{aligned} \tag{35}$$

The equal time (radial) canonical commutators in bulk satisfy:

$$\begin{aligned}
&\Big[\;\rho^a(z)\ ,\ \rho^b(z')\;\Big] = 0\ ,\\
&\Big[\;P^a(z)\ ,\ P^b(z')\;\Big] = 0\\
\text{and}\qquad &\Big[\;\rho^a(z)\ ,\ P^b(z')\;\Big] = i\eta^{ab}\;\delta(z - z')\ .
\end{aligned} \tag{36}$$

Since the zero-modes have been separated out from the string coordinates, the integral

$$\int_{\partial\Sigma} d\theta\;\rho^i(z) \;=\; 0\ .$$

The commutator (36) can be substituted for the conjugate momenta $P^b(z)$ and at the disk boundary becomes

$$\Big[\ \rho^a(\theta)\ ,\ \rho^b(\theta)\ \Big]_{\partial\Sigma} = 0\ . \tag{37}$$

However, the presence of zero-modes $\rho^a_{(0)}$, in an orthonormal moving frame, contribute towards the non-vanishing of commutator (37) and thus the open string centre of mass becomes non-commutative in presence of the background fields. Since the dynamics on the world-volume of a D_p-brane is due to the string fluctuations at its end points, the commutator for the D_p-brane collective coordinates in a static gauge can be expressed as

$$\Big[\ f^a(t(\theta),\sigma_i(\theta))\ ,\ f^b(t(\theta),\sigma_j(\theta))\ \Big]_{\partial\Sigma}\ =\ \pm 2\pi\ i(M^{-1}[B+\bar{F}])^{ab}\ , \tag{38}$$

where $M_{ab} = \eta_{ab} - [B+\bar{F}]_a^c\ [B+\bar{F}]_{cb}$. The right hand side in eq.(38) is due to the induced fluxes on the world-volume of a D_p-brane leading to the non-commutative geometry. From the string theory perspective, the non-commutative geometry is a boundary phenomena and thus closed strings do not perceive this special geometry. However, for a D_p-brane the non-commuatative feature is in bulk which represents its world-volume.

Now generalizing the Lorentz rotation (R)-matrix from our earlier discussions [11], the non-trivial components of the non-zero modes can be re-written as $\tilde{\rho}^a = R^a_b\ \rho^b$. Then, the boundary conditions for the (tilted) D_p-brane, in terms of the transformed coordinates, become

$$\begin{aligned}
\partial_{e\pm}\ \tilde{\rho}_i\ &=\ 0\\
\partial_{m_p\pm}\ \tilde{\rho}_{(p)i}\ &=\ 0\\
\text{and}\qquad\qquad \tilde{\rho}_\alpha\ &=\ 0\ ,
\end{aligned} \tag{39}$$

where $\partial_{e\pm} = [\partial_n\ \pm i\ \mathcal{N}_{(0)}\mathcal{N}_{(i)}\ E_i\partial_\theta]$ lie on the $(0i)$-planes and are defined with respect to the boost from the original frame. The remaining components are magnetic in nature and are used to rotate the (ij)-planes: $\partial_{m_p\pm} = [\partial_n\ \pm i\ \mathcal{N}_{(i)}\mathcal{N}_{(j)}\ (B_{ij}+\bar{F}_{ij})\partial_\theta]$. Here the transformed boundary conditions for the tilted D_p-brane in the tangential directions $(\tilde{\rho}_i, \tilde{\rho}_{(p)i})$ are the usual Neumann conditions. At the first sight, the conditions (39) appear to identical with a new D_p-brane in absence of the electric and magnetic fluxes. Nevertheless, the fluxes are manifested to rotate the original D_p-brane to a tilted (new) one and the transformed tangential coordinates, $\tilde{\rho}_a$, become non-local. Thus the non-locality depends on the electric and magnetic fluxes and the explicit

R-matrix[7] can be given by

$$R_{ab} \ = \left(\frac{1 - \mathcal{N}(a)\mathcal{N}_{(b)}\,(B + \bar{F})}{1 + \mathcal{N}_{(a)}\mathcal{N}_{(b)}\,(B + \bar{F})} \right)_{ab} \tag{40}$$

It is straight forward to note that the equal time commutator for the string modes (including the zero modes) becomes a delta function along the remaining spatial coordinates. This provides an explicit demonstration that the non-commutativity on the world-volume of a D_p-brane can be manifested as the non-locality on its world-volume.

8 Generalization to super D_p-branes

The D_p-brane dynamics obtained (33) in presence of arbitrary string backgrounds may be generalized to include the respective fermionic partners for the open string coordinates $X^\mu(z,\bar{z})$[8] as well as for the collective coordinates $f^\mu(t,\sigma_i)$. In general, the super-symmetric D_p-brane analysis becomes highly non-trivial in presence of super-string backgrounds, mostly due to the D_p-brane collective coordinates, $f^\mu(t,\sigma_i)$, and the $U(1)$ gauge field, A^a, living on its world-volume. However, for a constant $U(1)$ gauge field strength F_{ab}, the computations simplify to some extent in an orthonormal frame and the effective dynamics for a super D_p-brane may be addressed with super-string background fields.

As a first step, towards a generalization of the D_p-brane effective dynamics to the case of the super D_p-brane, one needs to consider the type I super-string.[9] In fact, type I open super-string is un-oriented and thus can be considered as the interacting open and closed strings ($B = 0$). In this section, we briefly present the computations for the fermionic partners by drawing analogy from that of the bosonic case (with vanishing KR potential) to avoid any repetition. This is indeed the case for D_p-branes with $p = 0, 2, 4, \ldots$ in type IIA theory.

Now re-consider the non-linear sigma model (2) with the Majorana fermions $\psi^\mu(z)$ as the super-partners of the (non-oriented) open string coordinates $X^\mu(z,\bar{z})$. Also, consider $\chi^\mu(t,\sigma_i)$ as the fermionic coordinates for the D_p-brane. For simplicity, the zero-modes are separated out from the fermionic coordinates similar to the one under the bosonic discussions. In an orthonormal frame, with a conformal gauge, the non-zero fermionic modes in the non-linear

[7]The modified Neumann matrix (18) can be tuned to the usual disk propagator by the R-matrix at the expense of the electric and magnetic fluxes.

[8]Here the space-time index is uderstood with ($\mu = 0, 1, 2 \ldots 9$).

[9]In the bosonic case, we have considered oriented ($B \neq 0$) open string to obtain the effective dynamics of a D_p-brane. It allows one to analyze the type II ($N = 2, D = 10$) close super-string channel. However, a consistent open super-string generalization reduces the super-symmetry to $N = 1$ (type I).

sigma model action[10] in the Neveu-Schwarz-Ramond (NSR) formalism can be expressed as

$$S(\psi, A) \;=\; \frac{i}{4\pi\alpha'}\Big(\;-\; \int_{\Sigma} d^2z \;\psi_A \partial\psi^A \;+\; \mathcal{N}_{(A)}\mathcal{N}_{(B)}\; \bar{F}_{AB} \int_{\partial\Sigma} d\theta \;\; \psi^A \;\psi^B \Big) \;, \tag{41}$$

where $\psi^A \;=\; \hat{e}^A_\mu \psi^\mu$. Then the expression (41) along with that in eq.(13) for vanishing KR potential ($B = 0$), takes care of the interacting type I open super-string with the massless closed string fields. The complete action (eqs.(13) and (41)) can be seen to be invariant under the super-symmetric transformations

$$\begin{aligned} \delta\rho^A \;&=\; \bar{\epsilon}\psi^A \\ \text{and} \qquad \delta\psi^A \;&=\; -i\epsilon\gamma^{\bar{a}}\partial_{\bar{a}}\rho^A \;, \end{aligned} \tag{42}$$

where ϵ denotes an infinitesimal (constant) Majorana spinor and $\gamma^{\bar{a}}$ denotes two-dimensional matrices on the string world-sheet. In addition, the covariant condition (1) is also modified due to the fermions at the disk boundary and can be expressed as a constraint.

Now the path-integral (3) can be generalized appropriately by taking into account the proper fermionic measures. The effective dynamics for the non-zero modes can be described manifestly in super-field notations [26] and in a static gauge takes the form:

$$\hat{\mathcal{S}}(f, \chi)_{SUSY} = \frac{1}{g_s} \int \;\hat{\mathcal{D}}\hat{X}^A(z, \phi)\; \hat{\mathcal{D}}\hat{\sigma}_a(\theta)\; \delta(\hat{X}^A - \hat{f}^A)_{\partial\Sigma}\; \exp\Big(- \hat{S}[X, \psi]_{SUSY}\Big) \;, \tag{43}$$

where $\hat{\mathcal{D}} = \partial_\phi - \phi\partial_z$ denotes the super derivative ($\hat{\mathcal{D}}^2 = \partial_z$) and $(\phi, \bar{\phi})$ correspond to the super-symmetric partners of $(z, \bar{z})$. The collective coordinates for the super D_p-brane is represented by $\hat{f}^\mu(\sigma_a, \beta_a)$, where β_a denotes the super-partners of σ_a. The super-space string coordinates become

$$\hat{X}^A(z, \phi) = X^A(z) + \phi\; \psi^A(z) \tag{44}$$

and the corresponding measure in the path-integral (43) takes the form:

$$\hat{\mathcal{D}}\hat{\sigma}_a = \mathcal{D}\sigma_a \mathcal{D}\beta_a \;.$$

The super-space action in eq.(43) can be expressed as

$$\hat{S}[\rho, \psi]_{SUSY} = S[X, A, B]_{(B=0)} + S[\psi, A] \tag{45}$$

and corresponds to an interacting type I super-string dynamics at low energy. It is straight forward to note that the dynamics of the D_p-branes is due to the underlying type I super-string

[10] We follow the conventional notations as in ref.[25].

coupled to the curved backgrounds of closed string. The additional boundary condition arises from the fermions in the NS and Ramond sectors. For the non-zero modes, they can be given by

$$\begin{aligned} & \left(\psi^a_+ \mp i\psi^a_-\right) \ - \ \mathcal{N}_{(a)}\mathcal{N}_{(b)} \ \bar{F}^a_b\left(\psi^b_+ \pm i\psi^b_-\right) \ = \ 0 \\ \text{and} \qquad & \left(\psi^\alpha_+ \ \pm \ \psi^\alpha_-\right) \ = \ 0 \ , \end{aligned} \tag{46}$$

where $\psi^A_\pm$ are the left and right moving components of ψ^A on the string world-sheet. The first expression in eq.(46) is the modified Neumann condition in presence background fields where as the second expression there represents the Dirichlet boundary condition.

The induced super-symmetric invariant metric, $\hat{h}_{ab}$, and the RR $(p+1)$-form, $\hat{C}_{abc...}$, are modified appropriately due to the fermionic coordinates, $\chi^A(\sigma_a)$, in a static gauge, on the world-volume of the D_p-brane. The super-symmetric (invariant) fields can be expressed in the orthogonal frame and they take the form

$$\begin{aligned} & \hat{h}_{ab} = \eta_{AB} D_a \hat{f}^A D_b \hat{f}^B \\ \text{and} \qquad & \hat{C}_{abc...} = \hat{C}_{ABC...} D_a \hat{f}^A D_b \hat{f}^B D_c \hat{f}^C \ldots \ , \end{aligned} \tag{47}$$

$$\begin{aligned} \text{where} \qquad & \hat{f}^A(\sigma_a, \beta_a) = f^A(\sigma_a) + \beta_a \chi^A(\sigma_a) \\ \text{and} \qquad & D_a = \partial_{\beta_a} - \beta_a \partial_a \ . \end{aligned} \tag{48}$$

Now the path-integration over the fermionic modes (ψ^a, ψ^α) can be performed by extending the analysis for the bosonic case discussed in previous chapters 5 and 6. The calculation essentially reduces to that of Jacobian involving the determinants. The path-integral in bulk becomes Gaussian and the Jacobian due to the fermions in the NS-NS sector is found to be unity. Since the field contents in the NS-NS sector turns out to be similar to that of the bosonic string, the Jacobian for the bosons in this sector becomes the one obtained (24) in the bosonic case with vanishing KR potential. On the other hand, in the RR sector the Jacobian due to the fermions exactly cancels that of the bosons. There is no substantial contribution to the Jacobian in the RR sector. Thus, the presence of fermions give a trivial contribution to the effective dynamics and do not affect the result obtained in bulk for the bosonic case. In fact, the super-string effective action takes the same form as the bosonic string theory though the field contents are modified with respect to the fermionic partners.

On the other hand, the path-integral over the boundary fields involves the fermionic partners and has to be evaluated perturbatively. The non-triviality is mainly due to the super D_p-brane collective coordinates $\hat{f}^\mu(\sigma_a(\theta), \beta_a(\theta))$. To calculate the boundary contribution, we consider the geodesic expansion[11] about a point (σ_a, β_a) in super-space [26]. As a consequence, the computations become non-trivial due to the modified induced fields (47) on the world-volume. The expression for the generalized extrinsic curvature, $\hat{K}$, can be obtained readily from the modified fields. After a considerable computations, the boundary path-integrals can be evaluated.[12] Final result of the path-integrations in bulk as well as at the boundary can be encoded in the generalized DBI action describing the super D_p-brane[13] $(p = 0, 2, 4 \ldots)$ dynamics to $\mathcal{O}(\alpha')$ in open string theory and becomes

$$\hat{\mathcal{S}}(f, \chi)_{SUSY} = T_p \int d^{p+1}\hat{\sigma} \sqrt{-\det\Big(\hat{h} + \bar{F}\Big)} + Q_p \int d^{p+1}\hat{\sigma}\ e^{\bar{F}} \wedge \hat{C}\ . \qquad (49)$$

9 Discussion

To summarize, the computations were performed in two parts, namely: for the bosonic D_p-brane and then the super D_p-brane. In the first part, we have demonstrated an explicit computations for an arbitrary D_p-brane (any $p = 0, 1, 2, \ldots$) in generic close string backgrounds to obtain the non-linear dynamics in open bosonic string theory. It was shown that the path-integration in bulk is essentially responsible for the non-perturbative dynamics. On the other hand, the integrations over the boundary fields were performed next to the leading order in α'. The perturbative corrections were expressed in terms of the induced curvatures on the world-volume of the D_p-brane and were found to be associated with a logarithmic divergence. A suitable re-normalization of the D_p-brane collective coordinates, $f^\mu(t, \sigma_i)$, were performed to obtain the non-linear DBI action describing the non-perturbative dynamics of the D_p-brane. In addition, the boundary conditions for the arbitrary D_p-brane, in presence of close string backgrounds, were re-viewed with the zero-modes. In a static gauge, the D_p-brane collective coordinates, $f^\mu(t, \sigma_i)$ were found to be non-commutative and the world-volume becomes non-local at the expense of non-commutative geometry. Further analysis of the world-volume geometry is believed to enhance the understanding of gauge theory and would be addressed else where.

In the second part, our computations were generalized to include the appropriate super partners in the type I super-string theory. The choice of orthonormal moving frame facilitates the inclusion of fermions in the locally inertial frame. The non-perturbative part of the

[11]It can be expressed from that of the bosonic one (8) by replacing the partial derivatives, ∂_a, with the super-derivatives, D_a, as in eq.(48) and taking a note on the super-coordinates $\hat{f}^\mu(\sigma_a, \beta_a)$.

[12]We skip the details of the steps involving re-normalization of the super-brane coordinates, which can also be intuitively obtained from that of the bosonic case discussed.

[13]The even 'p' case is straightforward to confirm from the eq.(49).

super D_p-brane dynamics was obtained by performing the Gaussian path-integration in bulk. The boundary integrals were evaluated perturbatively and the corrections in α' were found to be associated with the derivative expansion of the D_p-brane super-coordinates $\hat{f}^{\mu}(\sigma_a, \beta_a)$. An qualitative analysis for the boundary integrals were discussed by drawing analogy from its bosonic counterpart. Finally, the non-perturbative super D_p-brane dynamics were encoded in the non-linear dynamics of the DBI action.

In this paper, we have computed the disk amplitude and thus the $U(1)$ gauge field on the world-volume is consistent. However, at the quantum level (higher string loops), the consistency condition from the anomaly cancellation of interacting open super-string would require the group to be $SO(32)$. The issue of non-abelian world-volume dynamics becomes technically difficult and some attempts can be found in ref.[7]. Another important issue is the special world-volume geometry due to the zero-modes. From the string theory point of view, this is a boundary phenomena. However for a D_p-brane, the non-commutative feature is in the bulk of its world-volume. Since the world-volume theory for a D_p-brane is a dimensionally reduced super Yang-Mills to $(p+1)$-dimensions, the gauge theory becomes a non-local field theory and is a subtle issue.

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References

[1] J. Polchinski, TASI Lectures on D-branes, hep-th/9611050; C. Bachas, Lectures on D-branes, hep-th/9806199.

[2] J. Polchinski, Phys. Rev. Lett. **75** (1995) 4724 (hep-th/9510017).

[3] I. R. Klebanov and L. Thorlacius, Phys. Lett. **B371** (1996) 51 (hep-th/9510200); C. Bachas, Phys. Lett. **B374** (1996) 37 (hep-th/9511043); C. G. Callan and I. R. Klebanov, Nucl. Phys. **B465** (1996) 473; (hep-th/9511173); C. Schmidhuber, Nucl. Phys. **B467** (1996) 146 (hep-th/9601003); M. R. Garousi and R. C. Myers, Nucl. Phys. **B475** (1996) 193 (hep-th/9603194); W. Fischler, S. Paban and M. Rozali, Phys. Lett. **B381** (1996) 62 (hep-th/9604014); A. Hashimoto and I. R. Klebanov, Phys. Lett. **B381** (1996) 437 (hep-th/9604065); C. Callan and J. Maldacena, Nucl. Phys. **B513** (1998) 198 (hep-th/9708147); M. R. Garousi and R. C. Myers, Nucl. Phys. **B542** (1999) 73 (hep-th/9809100).

[4] J. Dai, R. G. Leigh and J. Polchinski, Mod. Phys. Lett **A4** (1989) 2073.

[5] R. G. Leigh, Mod. Phys. Lett **A4** (1989) 2767.

[6] M. R. Douglas, J.Geom.Phys. **28** (1998) 255 (hep-th/9604198).

[7] A. A. Tseytlin, Nucl. Phys. **B501** (1997) 41 (hep-th/9701125).

[8] A. A. Tseytlin, Nucl. Phys. **B524** (1998) 41 (hep-th/9802133).

[9] S. Hirano and Y. Kazama, Nucl. Phys. **B499** (1997) 495 (hep-th/9612064); Y. Kazama, Nucl. Phys. **B504** (1997) 285 (hep-th/9705111).

[10] S. Kar and Y. Kazama, Int. J. Mod. Phys. **A14** (1999) 1531 (hep-th/9807239).

[11] S. Kar, Nucl. Phys. **B** (1999) in press (hep-th/9812230).

[12] M. Cederwall, A. von Gussich, B. E. W. Nilsson, P. Sundell and A. Westerberg, Nucl. Phys. **B490** (1997) 179 (hep-th/9611159); E. Bergshoeff and P. K. Townsend, Nucl. Phys. **B490** (1997) 145 (hep-th/9611173; J. Aganagic, C. Popescu and J. H. Schwarz, Nucl. Phys. **B495** (1997) 99 (hep-th/9612080).

[13] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. **B160** (1985) 69; A. A. Tseytlin, Nucl. Phys. **B276** (1986) 391.

[14] C. G. Callan, C. Lovelace, C. R. Nappi and S. A. Yost, Nucl. Phys. **B308** (1988) 221.

[15] M. R. Douglas and C. Hull, J. High Energy Phys. **9802** (1998) 003 (hep-th/9711165).

[16] A. Connes, M. R. Douglas and A. Schwarz, J.High Energy Phys. **2** (1998) 003 (hep-th/9711162).

[17] M. R. Douglas, hep-th/9901146 (1999).

[18] M. Li, hep-th/9802052; M. Berkooz, Phys. Lett. **B430** (1998) 237 (hep-th/9802069); Y.-K. E. Cheung and M. Krogh, Nucl. Phys. **B528** (1998) 185 (hep-th/9803031); F. Ardalan, H. Arfaei and M. M. Sheikh-Jabbari, J.High Energy Phys. **9902** (1999) 016 (hep-th/9810072); C. Hofman and E. Verlinde, J.High Energy Phys. **9812** (1998) 010 (hep-th/9810116); C. Chu and P. Ho, Nucl. Phys. **B550** (1991) 151 (hep-th/9812219); M. Kato and T. Kuroki, J.High Energy Phys. **9903** (1999) 012 (hep-th/9902004).

[19] E. Witten, Nucl. Phys. **B460** (1996) 335 (hep-th/9510135).

[20] A. Abouelsaood, C. G. Callan, C. R. Nappi and S. A. Yost, Nucl. Phys. **B280** [FS18] (1987) 599; C. G. Callan, C. Lovelace, C. R. Nappi and S. A. Yost, Nucl. Phys. **B288** (1987) 525.

[21] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. **B163** (1985) 123.

[22] A. M. Polyakov, Phys. Lett. **B103** (1981) 207.

[23] M. R. Douglas, hep-th/9512077 (1999).

[24] L. Alvarez-Gaume, D. Z. Freedman and S. Mukhi, Ann.of Phys. (1981) 85; E. Braaten, T. L. Curtright and C. K. Zachos, Nucl. Phys. **B260** (1985) 630.

[25] M. B. Green, J. H. Schwarz and E. Witten, Superstring theory, Vol. I, Cambridge Univ. Press (1987).

[26] D. Friedan, E. Martinec and S. Shenkeer, Nucl. Phys. **B271** (1986) 93.